Sparsity-based super-resolution in instruments for diagnostics of short pulses

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Abstract: We demonstrate experimentally algorithmic super-resolution for diagnostics of short pulses (amplitude and phase). Our approach is based on using the measured data for finding a mathematical basis in which the pulse is represented compactly.

The resolution limits of instruments for diagnostics of short laser pulses are defined by their physical properties. For example, the spectral response functions of photodiodes exhibit a low-pass-filter form, with a characteristic cutoff-frequency f_c, where $f_c \sim 1/\tau_c$, τ_c being the response time of the photodiode that is associated with the transport time of the charge carriers in the material. In cross-correlation frequency resolved optical gating (X-FROG), the physical resolution is primarily limited by the width of the reference pulse (its amplitude and phase must be known in advance) and also by the width of the nonlinear crystal which determines its spectral response due to phase-matching effect. These physical resolution limits, however, do not take into account additional information about the structure of the measured pulse, which is often available as "prior information". For example, it is sometime known in advance that the internal structure of the intensity waveform exhibit several peaks with a limited range of time-scales. Naturally, using such prior information for algorithmic enhancement of the resolution beyond the physical limit of the device is highly desirable, as it could yield improved resolution for the most sophisticated measurement systems. Perhaps even more important, it could greatly improve instruments with lower inherent resolution, which are often much simpler, more compact, more robust and cheaper.

A related problem was tackled recently in imaging: it was proposed [1] and demonstrated [2] that knowing that some image is sparse in a known basis, enables algorithmic recovery of subwavelength features, which are otherwise lost because they are carried upon evanescent waves – never arriving at the detector. The underlying concept relies on minimizing the number of degrees of freedom: knowing that the sought information is sparse means that it projects onto a small number of terms of some (known) basis functions. This 'prior' provides an effective tool to find the correct extrapolation of the plane-wave components, and recover the information carried by evanescent waves.

Here, we propose and demonstrate experimentally the employment of sparsity-based concepts for enhancing the resolution of time-resolved instruments, significantly beyond their inherent physical limits. Our algorithm uses the measured data to find a proper basis for compact mathematical representation of the input signal, and then utilize it for extrapolating the resolution significantly beyond the inherent physical limit of the measurement device. We present experimental example of super-resolution in photodiodes, and a theoretical example of super-resolution in X-FROG. Our approach is general and can be implemented in all time-resolved devices, including diagnostic instruments of femtosecond and attosecond laser pulses.

We first present our experimental super-resolution in photodiodes. We constructed a laser pulse containing three peaks by splitting and later combining an uncompressed pulse from a Ti:Sapphire laser amplifier system into three routs with different lengths. Pulses with several peaks of similar widths can be represented compactly using a proper Gauss Hermite (GH) basis. The problem is that this basis is not known, hence, our reconstruction algorithm have to first identify this basis. Our reconstruction algorithm uses only the direct output signal of the photodiode and the prior knowledge that the sought signal is sparse in one (unknown) member of the infinite family of GH bases. Our sparsity-based reconstruction algorithm is based on the algorithms that were developed for sub-wavelength imaging [1,2]. In a nuts shell, it searches for the extrapolation which yields the sparsest positive spectrum in a basis that is known in advance. Our algorithm includes a stage that scan through an infinite family of bases to find the 'optimal' one in which the sought signal is represented most compactly.

The experimental results are displayed in Figure 1. The laser pulse is detected by a "slow" photodiode that is characterized by 1000ps rise-time and also by a "fast" photodiode (175ps rise time) – whose measured signal we use as a comparison (Fig. 1a). We first measure the temporal and spectral responses of the photodiodes by detecting the output for a 30fs pulse (Figs. 1b and 1c). Figure 1d shows the measurement taken by the slow and fast photodiodes, while their Fourier spectra are shown in Fig. 1e. We implement our reconstruction scheme on the detected signals from both the "slow" and "fast" detectors. For comparison, we also implement Wiener deconvolution on the two detected signals. The reconstructed intensity and spectral profiles are shown in Figs. 1f and 1g, respectively. As shown, the Wiener deconvolution reconstruction using the slow photodetector is very different form the Wiener deconvolution and the sparsity-based reconstructions using the "fast" photodiode. This large deviation shows that Wiener deconvolution reconstruction using the "slow" photodiode signal completely fails to reconstruct the correct profile. On the other hand, our sparsity-based reconstruction using the "slow" photodiode signal matches very well the sparsity-based and Wiener deconvolution reconstructions using the "fast" photodiode (compare the solid black curve with the solid blue and dash red curves in Figs. 1f and 1g). This correspondence shows that our sparsity-based reconstruction exhibits super-resolution, significantly better than Wiener deconvolution. Comparing the deviations in Fig. 1g, we conclude that our sparsity-based reconstruction has increased the resolution by a factor of ~5 over the Wiener deconvolution.

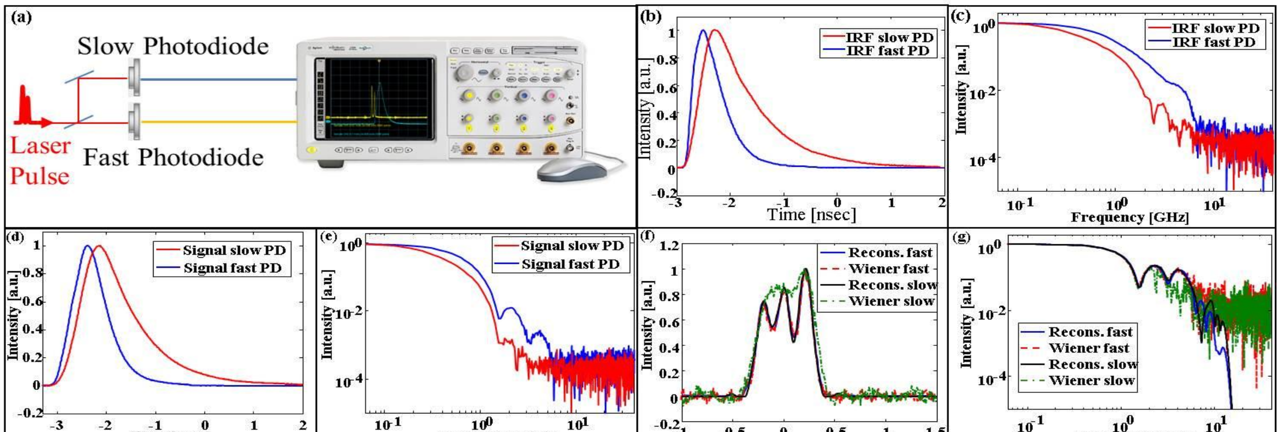

Figure 1: Experimental super-resolution of photodiode (a) Experimental setup: a laser pulse probed by slow (1000 ps rise-time) and fast (175 ps rise-time) photodiodes directly. (b) Measured impulse response functions of the slow photodiode (red) and the fast photodiode (blue). (c) Fourier spectrum of the impulse response functions shown in (b). Signals (d) and spectra (e) measured by the slow (red) and fast (blue) photodiodes. Reconstructed pulse-shapes (f) and their spectra (g) by implementing the our sparsity-based algorithm using the measured signals from the slow (solid black) and fast (solid blue) and reconstruction using Wiener de-convolution using the measured signals from the slow (dashed green) and fast (dashed red) photodiode. The sparsity-based reconstructed pulse-shape using the signal from the slow photodiode matches very well the measured pulse through Wiener de-convolution and sparsity-based using the signal from the fast photodiode.

Next, we present theoretical example of sparsity-based super-resolution in X-FROG. Here we introduced the sparsity prior that the amplitude of the pulse is sparse in GH basis into a standard iterative projection X-FROG algorithm [3]. Here, we use a relatively long reference pulse (Fig. 2a) and noisy measurement (spectrogram) (Fig. 2b) such that the reconstruction without sparsity (Fig. 2c) does not reconstruct the original pulse well (e.g. it misses the left dip). As clearly shown, the sparsity prior significantly improves the reconstruction.

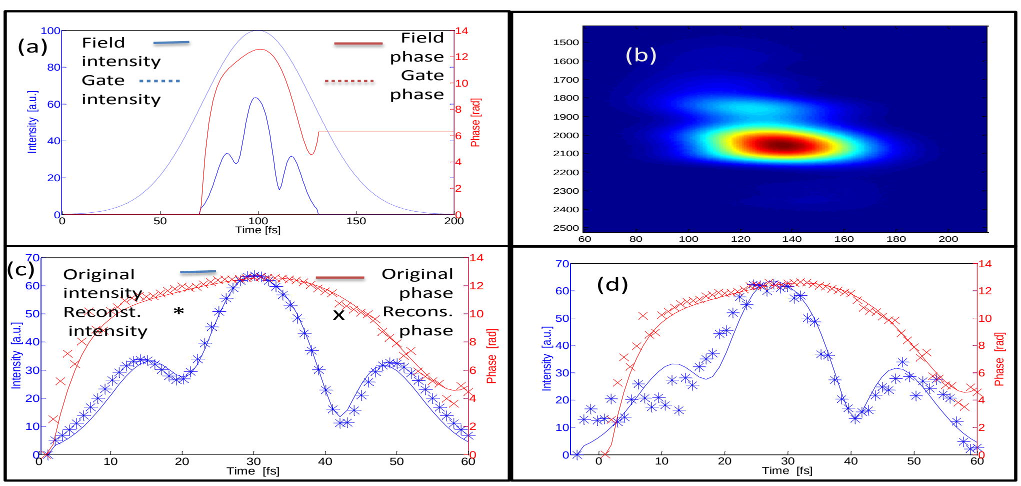

Figure 2: Theoretical super-resolution in cross-correlation frequency resolved optical gating (X-FROG) (a) Amplitudes and phases of the probed and reference pulses (b) calculated spectrogram (c) reconstruction w/o sparsity prior (d) Reconstruction with sparsity prior.

References

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